

SENSITIVITY ANALYSIS OF COMBINED STANDARD UNCERTAINTIES EVALUATED IN SPRTs' CALIBRATION ACCORDING TO THE ITS-90

Sonia GAIȚĂ

Romanian Bureau of Legal Metrology - National Institute of Metrology
Șos. Vitan-Bârzești 11, 75669 București, ROMÂNIA
E-mail: soniagaita@hotmail.com

Résumé

L'auteur a développé un modèle élargi pour l'évaluation de l'incertitude-type combinée (ITC) associée à la résistance des TRPE étalonnés aux points fixes de définition de l'EIT-90, en incluant toutes les grandeurs et tous les paramètres qui sont connus à présent de contribuer à l'ITC. L'utilisation du modèle dans le sous-domaine de l'EIT-90 allant de 273.15 K à 692.677 K est illustrée. Le poids de la contribution de chaque incertitude élémentaire à l'ITC est estimé. Des graphiques et des tableaux qui synthétisent les résultats de l'étude fournissent une base solide pour identifier les grandeurs d'entrée dont les incertitudes de mesure ont une contribution significative à l'ITC.

Abstract

The author developed a comprehensive model to evaluate the combined standard uncertainty (CSU) in the calibration of SPRTs at the defining fixed points of the ITS-90, by integrating all quantities and parameters presently known to contribute to the CSU. The paper illustrates the use of the model in the sub-range from 273.15 K to 692.677 K of the ITS-90. The weight of the contribution of each elementary uncertainty to the CSU was estimated. Charts and tables that summarise the results of the study provide a solid base for the identification of those input quantities whose uncertainties of measurement have a significant contribution to the CSU.

Introduction

After the new Temperature Scale [1] was adopted in 1990, and especially after the publication in 1993 of the Guide to the Expression of Uncertainty in Measurement [2], important efforts of researchers in thermometry were focused on evaluating the uncertainties of realization of the Scale. This work was boosted by the increase of the number of international comparisons aiming at evaluating the equivalence among national standards: without indicating the uncertainty associated with the results of measurements, those results cannot be compared among them. Moreover, the uncertainty evaluation methods should be consistent.

The studies published to date indicate that the highest interest in that respect has been raised by the assessment of the uncertainty associated with the calibration of an SPRT in compliance with ITS-90. The present paper relates to one [3] of those studies and proposes an enhanced model. The powerful and versatile features of the new model are illustrated by using it for the evaluation of the individual impact of each input quantity on the combined standard uncertainty, evaluated at fixed points and over the entire sub-range from 273.15 K to 692.677 K.

The mathematical model

The model functions

The aim is to find the analytical expression of the resistance of Standard Platinum Resistance Thermometers (SPRTs) at the temperature of a defining fixed point of the ITS-90 [1]. An essential feature of the model is that the corrections applied to the result of the measurement of that resistance are considered themselves measurands and the model function integrates their expanded expressions in (input) quantities that are known to determine their value.

Let R_{TPW} be the SPRT's resistance at the triple point of water (TPW) and R_{FP} the SPRT's resistance at any other fixed point (FP). The proposed model functions are then:

$$R_{TPW} = R'_{TPW} + C_{TPW} / K \quad R_{TPW} \left. \frac{dW_r}{dT_{90}} \right|_{T_{90}=(273.16\text{ K})} \quad (1)$$

and

$$R_{FP} = R'_{FP} + C_{FP} / K \quad R_{TPW} \left. \frac{dW_r}{dT_{90}} \right|_{T_{90}=T(FP)} \quad (2)$$

where the approximation $\left. \frac{dW}{dT_{90}} \right|_{T_{90}} \cong \left. \frac{dW_r}{dT_{90}} \right|_{T_{90}}$ was used,

with $\left. \frac{dW_r}{dT_{90}} \right|_{T_{90}}$ evaluated by means of equation (9a) or (10a)

in [1].

R'_{TPW} and R'_{FP} are the resistance of the SPRT at TPW and at FP, respectively, with the corrections and correction

factors included to compensate the following systematic effects involved in the resistance measurement:

- the self-heating effect;
- the temperature induced variation of the resistance of the standard resistor;
- the drift of the resistance of the standard resistor since its latest calibration;
- the non-linearity of the bridge, noise and so on.

Using the symbols:

$R_{s/TPW}$ – resistance of the standard resistor;

b_{TPW} – coefficient of the drift of the resistance of the standard resistor since its last calibration;

t_{TPW} – time of the calibration of SPRT at TPW;

t_0 – time of the calibration of the standard resistor ($t_0 = 0$);

$\alpha_{1/TPW}$, $\alpha_{2/TPW}$ – temperature coefficient of the standard resistor during the measurements using the currents I_1 and I_2 , respectively;

$T_{b1/TPW}$, $T_{b2/TPW}$ – temperatures of the oil bath where is maintained the standard resistor during the measurements using the currents I_1 and I_2 , respectively;

T_r – calibration temperature of the standard resistor ($T_r = 293.15$ K);

$r_{1/TPW}$, $r_{2/TPW}$ – ratios of the resistance measured using the currents I_1 and I_2 , respectively;

$r_{c1/TPW}$, $r_{c2/TPW}$ – correction factors for the readings of the bridge r_1 and r_2 , respectively;

$I_{1/TPW}$, $I_{2/TPW}$ – measurement currents;

the expression of R'_{TPW} are:

$$R'_{TPW} = \frac{R_{s/TPW} [1 + b_{TPW} (t_{TPW} - t_0)]}{I_{1/TPW}^2 - I_{2/TPW}^2} \cdot \{ [1 + \alpha_{1/TPW} (T_{b1/TPW} - T_r)] r_{1/TPW} r_{c1/TPW} I_{2/TPW}^2 - [1 + \alpha_{2/TPW} (T_{b2/TPW} - T_r)] r_{2/TPW} r_{c2/TPW} I_{1/TPW}^2 \} \quad (3)$$

and similarly for FP.

C_{FP} and C_{TPW} are the sum of the rest of corrections that have to be applied to the measurement result to compensate for:

- the effect of the hydrostatic pressure;
- the influences of the chemical impurities and of the variation of the isotopic composition of water, respectively;
- the effect of parasite heat fluxes (departure from thermal equilibrium);
- the deviation of the pressure of gas in the fixed point cell from the reference pressure [1].

Hence, the expressions of C_{FP} and C_{TPW} are:

$$C_{FP} = -A_{FP} h_{FP} + C_{1/FP} + C_{2/FP} - B_{FP} \delta p_{FP} \quad (4)$$

$$C_{TPW} = -A_{TPW} h_{TPW} + C_{1/TPW} + C_{2/TPW} \quad (5)$$

where:

A_{FP} , A_{TPW} – is the temperature variation with immersion depth h , at FP and TPW, respectively;

h_{FP} , h_{TPW} – immersion depth;

$C_{1/FP}$, $C_{1/TPW}$ – correction related to the influences of the chemical impurities and to the variation of the isotopic composition of water, respectively;

$C_{2/FP}$, $C_{2/TPW}$ – corrections of the effects of the parasite heat fluxes;

B_{FP} – temperature variation of the FP with the deviation of the pressure of the gas in the cell from the reference pressure;

δp_{FP} – correction of the deviation of the pressure of the gas in the cell from the reference pressure.

The model functions used to determine the characteristics $W = f(T_{90})$ of SPRTs, where $W(T_{90}) = R(T_{90})/R(273.16 \text{ K})$, are the interpolation equations given in ITS-90:

$$W(T_{90}) = W_r(T_{90}) + \Delta W(T_{90}) \quad (6)$$

where $W_r(T_{90})$ is a reference function and $\Delta W(T_{90})$ is a function with the coefficients calculated on the basis of values of the SPRT resistance determined at the fixed points.

The analytical expression of $W(T_{90})$ at any temperature within a particular sub-range of the ITS-90 will depend on the ratios $W_{FP} = R_{FP}/R_{TPW}$ at the defining fixed points of the ITS-90 in that sub-range and, eventually, on the input quantities involved in the evaluation of the pairs R_{FP} and R_{TPW} , determined at those fixed points.

The combined standard uncertainty (CSU)

As the calibration of an SPRT aims at determining its characteristic $W = f(T_{90})$, CSU of the resistances R_{FP} are not of interest. Nevertheless, CSU of R_{TPW} is useful for checking the stability of the SPRT.

CSU of the ratio W_{FP} is required for the international comparison of the national realizations of the ITS-90. It is evaluated using the model function:

$$W_{FP} = \frac{R_{FP}}{R_{TPW}} \quad (7)$$

by replacing R_{FP} and R_{TPW} with their expressions (1) and (2).

Within the measurement sub-range of the SPRT, its W will be a function of all the input estimates x_i of the ratios W_{FP} at the FPs specified in ITS-90 and of temperature:

$$W = f(x_i, T_{90}) \quad (8)$$

The combined standard uncertainty associated with W at any temperature T_{90} is determined by the law of propagation of uncertainty for correlated input quantities [2]:

$$u_c^2(W) = \sum_{i=1}^N \left[\frac{\partial f}{\partial x_i} \right]^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i) u(x_j) r(x_i, x_j) \quad (9)$$

where $r(x_i, x_j)$ are the correlation coefficients and $\partial f/\partial x_i$ are the sensitivity coefficients.

Several assumptions were used to handle the complex issue of correlations among input quantities.

- R_s , b , α are quantities not associated with an “intrinsic” uncertainty, but only with an “inherent” uncertainty (they materialize the same value, regardless the

measurement they refer to, at an unknown position within the uncertainty range); the respective values (e.g. b_{TPW1} , b_{TPW2} , ..., b_{FP1} , b_{FP2} , ...) are therefore fully correlated and all of them can be represented by the same symbol (b , in this case), regardless the measurement they refer to. Similar considerations apply to A_{TPW} .

- Due to the position of R_s in the model function, its contribution to the CSU vanishes.
- As the measurement at TPW is carried out immediately after the measurement at FP, $t_{FP} \approx t_{TPW}$ and the entire drift of the standard resistor can be reduced in the expression of the model function.
- T_{b1} , r_{1} , r_{c1} , T_{b2} , r_{2} , r_{c2} , I_1 , I_2 have an “intrinsic” uncertainty (they materialize potentially different values at different measurements, within uncertainty range). Therefore they will receive distinct notations for each measurement, in order to avoid their reduction in the symbolic manipulation phase of the process, and no mutual correlation will be considered for them, even if, occasionally, their estimates are equal.
- $I_{1/TPW}$ and $I_{2/TPW}$ and, respectively, $I_{1/FP}$ and $I_{2/FP}$ are correlated on technical grounds, and the correlation coefficient is reasonably well approximated by 1.

These assumptions are valid in the case of CSU evaluated for the W determined at any temperature within the SPRT measurement range.

Model implementation and application in the temperature sub-range from 273.15 K to 692.677 K

The model was implemented by means of a dedicated computer programme developed in a software environment with symbolic processing facilities. That allowed for more than 70 variables to be considered, with their respective correlations. The resulting analytical expressions of the sensitivity coefficients are much too large to be interpreted, presented, or handled other way but by electronic means. For quick reference, the model and its implementation were labeled CAM (an acronym for Comprehensive Analytical Model).

The application of the model is illustrated for the temperature sub-range from 273.15 K to 692.677 K, as most of the information available in the reference literature [3], [4], [5], [6], [7], [8], [9] is for that sub-range.

According to ITS-90, the calibration of an SPRT in this sub-range requires the measurement of its resistance at the TPW (273.16 K) and at the FPs of Sn (505.078 K) and Zn (692.677 K). The expression of the deviation function is:

$$W(T_{90}) - W_r(T_{90}) = a [W(T_{90}) - I] + [W(T_{90}) - I]^2 \quad (10)$$

with the reference function $W_r(T_{90})$ defined by (10a) of [1]. The values of a and b in (10) are derived from the values $W_{Zn} = W(692.677 \text{ K})$ (where $W_{Zn} = R_{Zn}/R_{TPW}$) and W_{Sn} (where $W_{Sn} = R_{Sn}/R_{TPW}$) determined by measurements carried out in the following sequence: R_{Zn} , R_{TPW1} , R_{Sn} , R_{TPW2} .

In Table 1 is presented the basic set of input data (labeled TEST) used in this material to illustrate the model. Values originate from NIM Bucharest research projects and from reference literature.

Table 1. TEST input data

| Quantity | Estimate | Uncertainty |
|--|---|---------------------------------------|
| R_s | 9,999 947 Ω | $3 \times 10^{-6} \Omega$ |
| b | $-5.48 \times 10^{-10} \text{ d}^{-1}$ | $1.92 \times 10^{-10} \text{ d}^{-1}$ |
| α | $16.36 \times 10^{-6} \text{ K}^{-1}$ | $6 \times 10^{-8} \text{ K}^{-1}$ |
| $T_{b1/Zn}; T_{b2/Zn}$ | 293.169 K | 0.007 K |
| $T_{b1/TPW1}; T_{b2/TPW1}$ | 293.171 K | 0.007 K |
| $T_{b1/Sn}; T_{b2/Sn}$ | 293.174 K | 0.007 K |
| $T_{b1/TPW2}; T_{b2/TPW2}$ | 293.175 K | 0.007 K |
| $r_{1/Zn}$ | 6.556 916 2 | 15×10^{-7} |
| $r_{2/Zn}$ | 6.556 947 4 | 15×10^{-7} |
| $r_{1/TPW1}$ | 2.552 565 4 | 3×10^{-7} |
| $r_{2/TPW1}$ | 2.552 595 9 | 3×10^{-7} |
| $r_{1/Sn}$ | 4.831 262 5 | 13×10^{-7} |
| $r_{2/Sn}$ | 4.831 294 4 | 13×10^{-7} |
| $r_{1/TPW2}$ | 2.552 565 6 | 3×10^{-7} |
| $r_{2/TPW2}$ | 2.552 596 2 | 3×10^{-7} |
| $r_{c1/Zn}; r_{c2/Zn}; r_{c1/TPW1};$ $r_{c2/TPW1}; r_{c1/Sn}; r_{c2/Sn};$ $r_{c1/TPW2}; r_{c2/TPW2}$ | 1.000 000 0 | 1×10^{-7} |
| $I_{1/Zn}; I_{1/TPW1}; I_{1/Sn};$ $I_{1/TPW2}$ | $1.000 \times 10^{-3} \text{ A}$ | $1.6 \times 10^{-5} \text{ A}$ |
| $I_{2/Zn}; I_{2/TPW1}; I_{2/Sn};$ $I_{2/TPW2}$ | $1.414 \times 10^{-3} \text{ A}$ | $1.6 \times 10^{-5} \text{ A}$ |
| A_{TPW} | $-0.73 \times 10^{-3} \text{ K m}^{-1}$ | $6 \times 10^{-5} \text{ K m}^{-1}$ |
| $h_{TPW1}; h_{TPW2}$ | $187 \times 10^{-3} \text{ m}$ | $3 \times 10^{-3} \text{ m}$ |
| A_{Zn} | $2.70 \times 10^{-3} \text{ K m}^{-1}$ | $6 \times 10^{-5} \text{ K m}^{-1}$ |
| h_{Zn} | $195 \times 10^{-3} \text{ m}$ | $3 \times 10^{-3} \text{ m}$ |
| A_{Sn} | $2.20 \times 10^{-3} \text{ K m}^{-1}$ | $6 \times 10^{-5} \text{ K m}^{-1}$ |
| h_{Sn} | $192 \times 10^{-3} \text{ m}$ | $3 \times 10^{-3} \text{ m}$ |
| $C_{1/TPW1}; C_{1/TPW2}$ | 0 K | $1 \times 10^{-4} \text{ K}$ |
| $C_{1/Zn}$ | 0 K | $7 \times 10^{-4} \text{ K}$ |
| $C_{1/Sn}$ | 0 K | $5 \times 10^{-4} \text{ K}$ |
| $C_{2/TPW1}; C_{2/TPW2}$ | 0 K | $0.5 \times 10^{-4} \text{ K}$ |
| $C_{2/Zn}$ | 0 K | $2 \times 10^{-4} \text{ K}$ |
| $C_{2/Sn}$ | 0 K | $2 \times 10^{-4} \text{ K}$ |
| B_{Zn} | $4.3 \times 10^{-8} \text{ K Pa}^{-1}$ | $6 \times 10^{-10} \text{ K Pa}^{-1}$ |
| B_{Sn} | $3.3 \times 10^{-8} \text{ K Pa}^{-1}$ | $6 \times 10^{-10} \text{ K Pa}^{-1}$ |
| δp_{Zn} | 0 Pa | 100 Pa |
| δp_{Sn} | 0 Pa | 100 Pa |

In Fig. 1, one can see a representation of the CSU at any intermediate temperature, propagated from the calibration uncertainties at the fixed points, as determined with CAM and based on TEST input data in Table 1. Two cases are presented: when all previously specified correlations are considered (“CAM correlated” curve) and when no correlation is considered (“CAM uncorrelated” curve). The contribution of the correlation terms (especially that between I_1 and I_2) is proven to be significant (approx. 27% of $u_c(W) - \text{CAM correlated}$, at T_{Zn} and T_{Sn}). On the same chart there are drawn two curves determined with the model presented in [3]: when the input data from [3] was used (“[3]” curve) and when the input data from Table 1 was used (“[3] with TEST input data” curve); in this latest case, maximum differences against the “CAM correlated” curve are of 11% of $u_c(W) - \text{CAM correlated}$, at 350 °C.

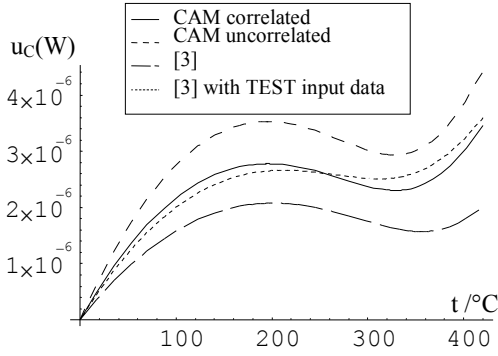


Figure 1. Propagated combined standard uncertainties

It is worth highlighting that the model function in CAM has other enhanced features as compared to the model presented in [3], besides the inclusion of correlation effects. Two additional input quantities are considered in CAM: the correction for the influence of the parasite heat fluxes and the correction for the deviation of the gas pressure in the cell from the reference pressure. The impact of the variances associated with the estimates of these quantities on the CSU will be discussed further. The entire analysis that follows was carried out on the TEST set of input data.

The sensitivity analysis

The percentage ratio between the term $[(\partial f/\partial x_i)^2 u^2(x_i)]$ in (9) and the combined variance $u^2_c(W)$ will be termed in this material “the weight of the contributory variance (WCV) associated with the input estimate x_i in the combined variance $u^2_c(W)$ ”. In order to emphasize the weight of the contribution of each group of correlated input estimates, the denomination was extended to the ratio

between the term $\left[\sum_{i=1}^m \frac{\partial f}{\partial x_i} u(x_i) \right]^2$ and $u^2_c(W)$, where m is the

number in a group of mutually correlated input estimates, with unit correlation coefficients. For economy of notation, the same symbol is used below for the input estimates and for input quantities.

In Fig. 2 and Fig. 3 charts were drawn for the WCVs associated with R_{TPW} and, respectively, with W_{Zn} and W_{Sn} . $I_1 I_2$ is a symbol used for the group of correlated quantities I_1 and I_2 .

In the R_{TPW} case, dominates the WCV corresponding to $C_{1/TPW}$ applied to the variation of the isotopic composition of the water in the cell. In the W_{Zn} and W_{Sn} cases, the major contribution corresponds to the influence of the impurities present in the metal ($C_{1/Zn}$ and $C_{1/Sn}$), but the influence of $C_{1/TPW}$ is considerable here as well.

The WCV of R_s is almost 20% of the combined variance associated with R_{TPW} . Increasing the calibration accuracy of the standard resistor can reduce it. In W_{Zn} and W_{Sn} cases, the influence of R_s is null.

The experimental variances associated with r_1 and with r_2 characterizes the variability of the observed values; their WCV are big, but they can not be diminished, as $u^2(r_1)$ and $u^2(r_2)$ reflect the random variations of the influence quantities.

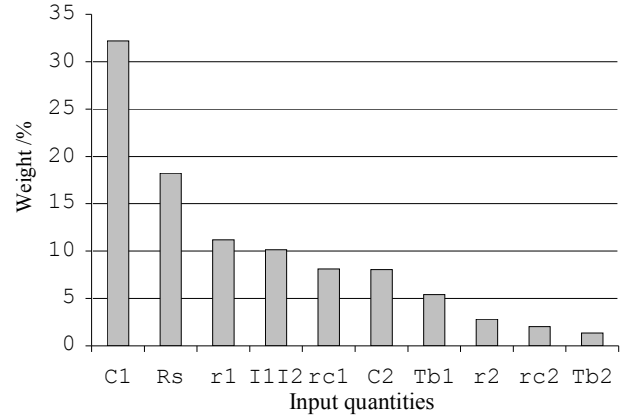


Figure 2. Weights of the contributory variances in the combined variance $u^2_c(R_{TPW})$

Significant are also the WCVs associated with the estimates of:

- the correlated quantities I_1 and I_2 and the correction factor r_c (determined by the technical performance of the bridge);
- the corrections of the effects of the parasite heat fluxes (departure from thermal equilibrium);
- the temperature of the standard resistor during measurement.

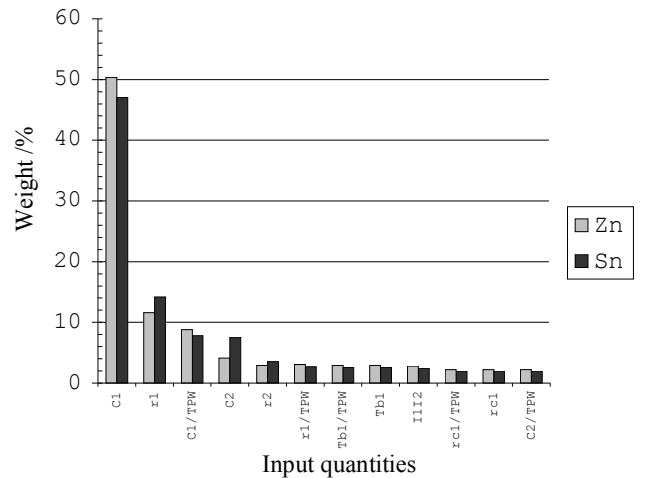


Figure 3. Weights of the contributory variances in the combined variance $u^2_c(W_{FP})$

The other input quantities have a small contribution to the combined variance associated with the output estimate.

For eight input quantities, having the greatest influence over W , the variation with temperature of their contributory variances was illustrated in Fig. 4 and 5, and the variation with temperature of their associated WCVs, in Fig. 6 and 7.

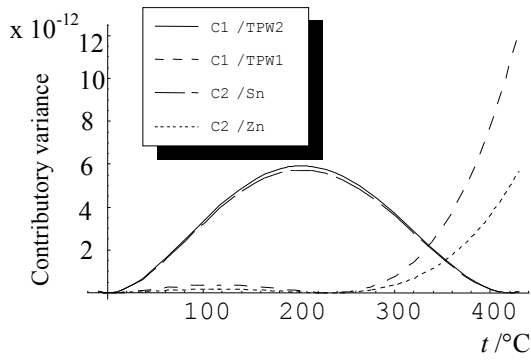


Figure 4. Contributory variances over temperature sub-range

As the contributory variance at an intermediate temperature derives from the propagation of the estimated contributory variance at a FP, it reaches a peak value in the neighborhood of the temperature of that FP and it is null at the temperature of the other FP (Fig. 4 and Fig. 5).

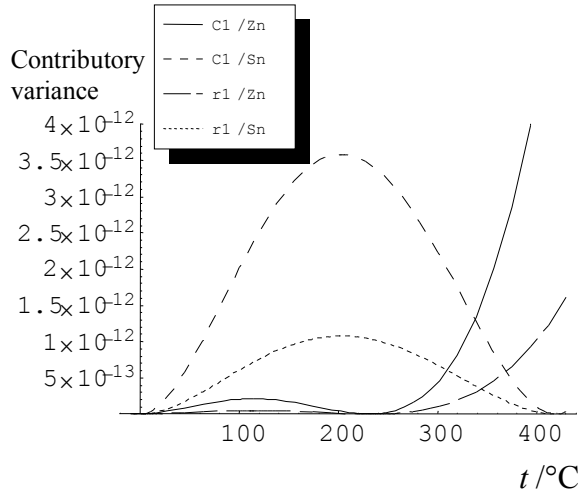


Figure 5. Contributory variances over temperature sub-range

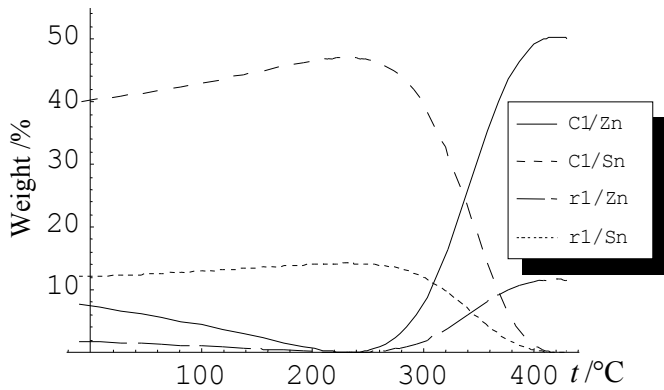


Figure 6. Weights of contributory variances over temperature sub-range

It is noteworthy that WCVs associated with the input estimates specific for W_{Sn} are larger at the temperature of WTP than the similar ones for W_{Zn} . That is due to the different positions of the temperatures of the two FPs relative to the limits of the sub-range we refer to (Fig. 6 and Fig. 7).

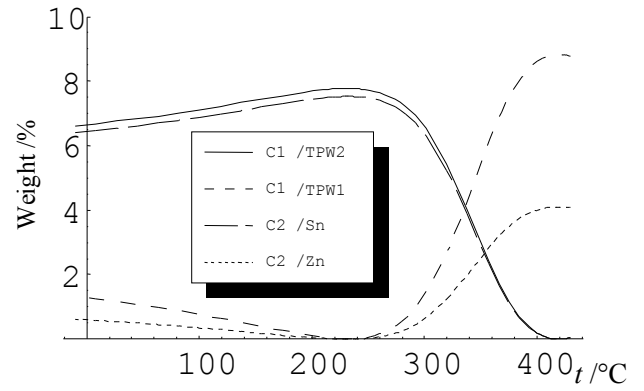


Figure 7. Weights of contributory variances over temperature sub-range

The list of values for the WCVs computed with CAM on the basis of the TEST input data is presented in Table 2, at the temperatures of the three defining fixed points of the ITS-90 in the sub-range and at 350 °C. The corresponding combined variances are listed in Table 3.

Table 2. Weights of contributory variances at various temperatures

| Quantity | Weight /% of contributory variance at : | | | |
|------------------|---|----------|----------|-------------------|
| | t_{TPW} | t_{Sn} | t_{Zn} | $t=350\text{ °C}$ |
| R_s | 0.00 | 0.00 | 0.00 | 0.00 |
| b | 0.00 | 0.00 | 0.00 | 0.00 |
| a | 0.00 | 0.00 | 0.00 | 0.00 |
| A_{TPW} | 0.00 | 0.09 | 0.11 | 0.21 |
| $I_1 I_2 / TPW1$ | 0.96 | 0.00 | 2.78 | 1.72 |
| $I_1 I_2 / Zn$ | 0.15 | 0.00 | 0.44 | 0.27 |
| $I_1 I_2 / TPW2$ | 1.60 | 2.44 | 0.00 | 0.93 |
| $I_1 I_2 / Sn$ | 0.49 | 0.75 | 0.00 | 0.29 |
| $T_{b1} / TPW1$ | 1.01 | 0.00 | 2.91 | 1.79 |
| $T_{b2} / TPW1$ | 0.25 | 0.00 | 0.73 | 0.45 |
| $r_1 / TPW1$ | 1.06 | 0.00 | 3.06 | 1.89 |
| $r_{c1} / TPW1$ | 0.77 | 0.00 | 2.22 | 1.37 |
| $r_2 / TPW1$ | 0.26 | 0.00 | 0.77 | 0.47 |
| $r_{c2} / TPW1$ | 0.19 | 0.00 | 0.55 | 0.34 |
| h_{TPW1} | 0.00 | 0.00 | 0.00 | 0.00 |
| $C_1 / TPW1$ | 3.05 | 0.00 | 8.82 | 5.44 |
| $C_2 / TPW1$ | 0.76 | 0.00 | 2.21 | 1.36 |
| $T_{b1} / TPW2$ | 1.68 | 2.56 | 0.00 | 0.98 |
| $T_{b2} / WTP2$ | 0.42 | 0.64 | 0.00 | 0.24 |
| $r_1 / TPW2$ | 1.77 | 2.70 | 0.00 | 1.03 |
| $r_{c1} / TPW2$ | 1.28 | 1.96 | 0.00 | 0.75 |
| $r_2 / TPW2$ | 0.44 | 0.68 | 0.00 | 0.26 |
| $r_{c2} / TPW2$ | 0.32 | 0.49 | 0.00 | 0.19 |
| h_{TPW2} | 0.00 | 0.00 | 0.00 | 0.00 |
| $C_1 / TPW2$ | 5.10 | 7.78 | 0.00 | 2.98 |
| $C_2 / TPW2$ | 1.27 | 1.94 | 0.00 | 0.74 |
| T_{b1} / Zn | 1.01 | 0.00 | 2.91 | 1.79 |
| T_{b2} / Zn | 0.25 | 0.00 | 0.73 | 0.45 |
| r_1 / Zn | 4.01 | 0.00 | 11.61 | 7.16 |
| r_{c1} / Zn | 0.77 | 0.00 | 2.22 | 1.37 |
| r_2 / Zn | 1.00 | 0.00 | 2.90 | 1.79 |
| r_{c2} / Zn | 0.19 | 0.00 | 0.55 | 0.34 |
| A_{Zn} | 0.00 | 0.00 | 0.01 | 0.01 |

Table 2 (continued)

| Quantity | Weight /% of contributory variance at : | | | |
|-----------------|---|----------|----------|---------------------------------|
| | t_{TPW} | t_{Sn} | t_{Zn} | $t=350\text{ }^{\circ}\text{C}$ |
| h_{Zn} | 0.00 | 0.00 | 0.01 | 0.00 |
| $C_{1/Zn}$ | 17.40 | 0.00 | 50.33 | 31.02 |
| $C_{2/Zn}$ | 1.42 | 0.00 | 4.11 | 2.53 |
| B_{Zn} | 0.00 | 0.00 | 0.00 | 0.00 |
| δp_{Zn} | 0.00 | 0.00 | 0.00 | 0.00 |
| $T_{b1/Sn}$ | 1.68 | 2.57 | 0.00 | 0.98 |
| $T_{b2/Sn}$ | 0.42 | 0.64 | 0.00 | 0.25 |
| $r_{1/Sn}$ | 9.28 | 14.16 | 0.00 | 5.42 |
| $r_{c1/Sn}$ | 1.28 | 1.96 | 0.00 | 0.75 |
| $r_{2/Sn}$ | 2.32 | 3.54 | 0.00 | 1.35 |
| $r_{c2/Sn}$ | 0.32 | 0.49 | 0.00 | 0.19 |
| A_{Sn} | 0.02 | 0.02 | 0.00 | 0.01 |
| h_{Sn} | 0.00 | 0.01 | 0.00 | 0.00 |
| $C_{1/Sn}$ | 30.83 | 47.04 | 0.00 | 18.00 |
| $C_{2/Sn}$ | 4.93 | 7.53 | 0.00 | 2.88 |
| B_{Sn} | 0.00 | 0.00 | 0.00 | 0.00 |
| δp_{Sn} | 0.00 | 0.00 | 0.00 | 0.00 |

Table 3. The combined variance associated with W

| | t_{TPW} | t_{Sn} | t_{Zn} | $t=350\text{ }^{\circ}\text{C}$ |
|------------|---------------------------|--------------------------|---------------------------|---------------------------------|
| $u_c^2(W)$ | 1.04688×10^{-22} | 7.3256×10^{-12} | 1.18934×10^{-11} | 5.55876×10^{-12} |

Obviously, the WCVs strongly depend on the actual values of the standard uncertainties associated with the input estimates. The weight and importance of these estimates can switch places when those uncertainties vary. That is easily revealed by CAM, and an example is presented in Fig. 8.

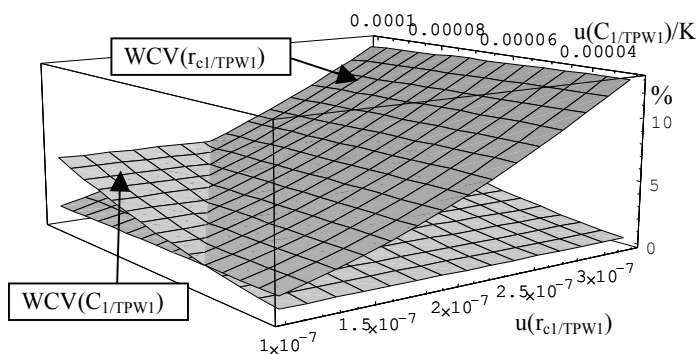


Figure 8. Weights of contributory variances as functions of standard uncertainties ($t = 350\text{ }^{\circ}\text{C}$)

Conclusions

The paper presented a comprehensive model for the evaluation of the combined uncertainty involved in the calibration of a SPRT according to the ITS-90.

The main features of the model were described, with highlights on the thorough consideration of correlations among input quantities. A brief comparison with results previously reported in the literature revealed significant impact of these correlations, as well as of the newly introduced influence quantities.

Selected results were presented to illustrate the capacity of the model to outline the input quantities that have the greatest contribution to the combined standard uncertainty. The model was proven as a resourceful tool, useful in a more efficient and focused effort to monitor and minimize the uncertainty of measurement results.

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